

Date _____
Page _____

* Stefan's law: - The experimental study of the rate of emission of heat energy by a hot body Tyndall helped Stefan (in 1879) to enunciate the law called Stefan's law. In 1884, Boltzmann gave a theoretical proof of Stefan's law on the basis of thermodynamics. Therefore, this law is also called Stefan-Boltzmann law.

According to this law, the rate of emission of radiant energy by unit area of a perfectly black body is directly proportional to the fourth power of its absolute temperature.

$$R \propto T^4$$

$$\text{or, } R = \sigma T^4 \quad \text{--- (i)}$$

Where σ is called the Stefan's constant. If the body is not perfectly black and its emissivity or relative emittance is e , then

$$R = e \sigma T^4 \quad \text{--- (ii)}$$

Hence e varies between zero and one, depending on the nature of the surface. For a perfectly black body $e=1$. The law is not only true for emission but also for absorption of radiant energy.

Hence if a perfectly black body at temperature T_1 is surrounded by a wall at a

temperature T_2 , the net rate of loss of heat energy per unit area of the surface is given by

$$R \propto (T_1^4 - T_2^4)$$

$$R = \sigma (T_1^4 - T_2^4) \quad \text{--- (ii)}$$

If the body has an emissivity e , then

$$R = e \sigma (T_1^4 - T_2^4) \quad \text{--- (iv)}$$

* Mathematical Derivation of Stefan's Law: -

The fact that black body radiations exert pressure similar to a gas, helps in applying thermodynamics to heat radiations.

Let ψ be the energy density of radiations inside a uniform temp. enclosure at temp T . P is the pressure and V is the volume.

Applying the first law of thermodynamics,

$$\delta H = dU + P \cdot dV \quad \text{--- (i)}$$

Applying thermodynamical relation,

$$\left(\frac{\partial H}{\partial V} \right)_T = \left(\frac{\partial P}{\partial T} \right)_V \quad \text{--- (vi)}$$

$$\left(\frac{\partial U + P \partial V}{\partial V} \right)_T = T \left(\frac{\partial P}{\partial T} \right)_V$$

$$\left(\frac{\partial U}{\partial V} \right)_T = T \left(\frac{\partial P}{\partial T} \right)_V - P \quad \text{--- (vii)}$$

$$\text{Now } U = V \psi$$

$$\text{and } P = \frac{\psi}{3}$$

$$\left[\frac{\partial U}{\partial V} \right]_T = \psi$$



Here ψ is a function of temp^r. alone.

Substituting these values in eqn. (vi), we get,

$$\psi = \frac{1}{3} \cdot \frac{d\psi}{dT} - \frac{\psi}{3}$$

$$\frac{4\psi}{3} = \frac{1}{3} \cdot \frac{d\psi}{dT}$$

$$\frac{d\psi}{\psi} = 4 \cdot \frac{dT}{T}$$

Integrating both sides,

$$\log \psi = 4 \log T + \text{constant}$$

$$\psi = kT^4 \quad \text{--- (viii)}$$

Here k is constant

Also, the total rate of emission per unit area of a black body is proportional to the energy density.

$$\therefore R \propto \psi \propto T^4$$

$$\therefore R = \sigma T^4 \quad \text{--- (ix)}$$

where σ is Stefan's constant.

The value of Stefan's constant in C.G.S. system is 5.672×10^{-5} C.G.S. unit, and in M.K.S. system, it is 5.672×10^{-8} M.K.S. units.